Midterm - 16/04/2025

Given Equations

Semiconductors at thermal equilibrium (Boltzmann and Fermi-Dirac formulas)

$n_0 = N_c \cdot e^{-\frac{E_C - E_f}{kT}}$
$p_0 = N_v \cdot e^{-\frac{E_f - E_V}{kT}}$
$n_0 = N_c \cdot \frac{1}{1+e^{\frac{E_C - E_f}{kT}}}$
$p_0 = N_v \cdot \frac{1}{1+e^{\frac{E_f - E_V}{kT}}}$
$n_i^2 = n \cdot p$
$n_i^2 = N_c N_v e^{-\frac{E_g}{kT}}$

Carrier transport

$\sigma = q \cdot (\mu_n n + \mu_p p)$
$L_n = \sqrt{D_n \tau_n}$
$L_p = \sqrt{D_p \tau_p}$

PN junction

$\phi_b = \frac{kT}{q} ln\left(\frac{N_a N_d}{n_i^2}\right)$
$\phi_n = \frac{kT}{q} ln\left(\frac{N_d}{n_i}\right)$
$\phi_p = -\frac{kT}{q} ln\left(\frac{N_a}{n_i}\right)$
$x_d(V) = \sqrt{\frac{2\epsilon_{Si}(N_a + N_d)}{qN_a N_d}(\phi_b - V)}$

MOS transistor

$V_{FB} = \phi_{ms} - \frac{qQ_{ss}}{C_{ox}}$
$V_{th} = V_{FB} - 2\phi_p + \gamma\sqrt{-2\phi_p}$
$\gamma = \frac{\sqrt{2\epsilon_{Si} q N_a}}{C_{ox}}$
$I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - \frac{V_{DS}}{2} - V_{th}\right)V_{DS}$

Given Constants

$k = 8.62 \cdot 10^{-5}\,[eV/K] = 1.38 \cdot 10^{-23}\,[J/K]$
$q = 1.60 \cdot 10^{-19}\,[C]$
$\epsilon_0 = 8.85 \cdot 10^{-14}\,[F/cm]$
$\phi_m(Al) = 3.2\,[V]$

Si properties

$n_i = 1.5 \cdot 10^{10}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$E_g = 1.12\,[eV] \quad @ \quad T = 300\,[K]$
$N_v = 1.04 \cdot 10^{19}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$N_c = 2.8 \cdot 10^{19}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$\chi_{Si} = 3.25\ eV$
$\epsilon_{Si} = 11.7 \cdot \epsilon_0$
$\epsilon_{SiO_2} = 3.9 \cdot \epsilon_0$

GaN properties

$E_g = 3.39\,[eV] \quad @ \quad T = 300\,[K]$
$N_v = 4.6 \cdot 10^{19}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$N_c = 2.3 \cdot 10^{18}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$

Exercise 01

Consider a sample of gallium nitride (GaN) in a wurtzite crystal structure. The valence and conduction bands effective density of states follow a $T^{3/2}$ thermal dependency law. At $0\,[K]$, the band gap energy is $E_g(0) = 3.47\,[eV]$. Assume that the energy gap depends on the temperature by this law:

$$E_g = E_g(0) - 7.7 \cdot 10^{-4} \cdot \frac{T^2}{T + 600} \tag{1}$$

- Calculate the intrinsic carrier concentration at $300\,[K]$.
- Calculate the intrinsic carrier concentration at $600\,[^\circ C]$: is this higher or lower than the room temperature value in silicon?
- Propose one application for this material, where silicon is inappropriate.

Exercise 02

Consider a sample of silicon (Si) at $300\,[K]$, doped with a concentration of boron (B) such that the Fermi level is $10\,[meV]$ higher than the dopant level. Consider a dopant ionization energy of $45\,[meV]$.

- Draw a band diagram of this sample of silicon, highlighting the zero-energy reference.
- Calculate the charge carrier concentration using both the Boltzmann approximation and the Fermi-Dirac distribution, and calculate the percentage error of the Boltzmann approximation over the full formula.
- Comment on the result: what is the condition that is not satisfied in this case for the use of the Boltzmann approximation?

Exercise 03

Consider a piece of lightly p-doped Si of length $1\,[cm]$ and section $1\,\left[mm^2\right]$ at $300\,[K]$. Upon application of $4\,[V]$ across the two extremities via ohmic contacts, a current of approximately $1\,[mA]$ is measured. Neglect any contact resistance.

- Based on the plot provided in figure 1, estimate the doping concentration.
- You want to modify the doping of this sample, in order to obtain a current one order of magnitude higher, either by increasing the B concentration, or by introducing some phosphorus (P). Which is more convenient to design? Give the required dopant concentration in the two cases.

Exercise 04

Consider an abrupt Si PN junction at $T = 300\,[K]$ with doping concentrations $N_a = 8 \cdot 10^{15}\,\left[cm^{-3}\right]$ and $N_d = 3 \cdot 10^{16}\,\left[cm^{-3}\right]$.

- Calculate the widths of the depleted regions in the p-side and n-side for the following cases: 1) thermal equilibrium; 2) $V_D = 0.5\,[V]$ (forward bias); 3) $V_D = -1\,[V]$ (reverse bias).
- Calculate and draw the space charge density $\rho(x)$ for the three cases.
- Calculate and draw the electric field $E(x)$ for the three cases. Indicate each time the value of E_{max} in $[V/cm]$.

Exercise 05

Consider the same junction as the previous exercise. The junction parameters are: $W_n = W_p = 150\,[um]$, $\tau_{n0} = \tau_{p0} = 1 \cdot 10^{-7}\,[s]$, $D_n = 27\,\left[cm^2/s\right]$, $D_p = 11\,\left[cm^2/s\right]$, $A = 1\,\left[mm^2\right]$.

- Check whether the device has short neutral sides or long neutral sides compared to the minority carriers diffusion lengths. Write the corresponding formula for the reverse saturation current I_S.
- Calculate I_S at the two temperatures $T_1 = 300\,[K]$ and $T_2 = 250\,[K]$. The valence and conduction bands effective density of states follow a thermal dependency $N_v \propto T^{3/2}$, $N_c \propto T^{3/2}$. Consider $E_g(250\,[K]) \approx E_g(300\,[K]) = 1.12\,[eV]$.
- Calculate $I(0.5\,[V])$ at T_1 and T_2.
- Draw in a single plot the $log|I(V)|$ curves at T_1 and T_2 and give a brief comparison of them.

Exercise 06

Consider a Si PN diode at $T = 300\,[K]$ with parameters: $N_a = N_d = 10^{15}\,\left[cm^{-3}\right]$, $I_S = 2 \cdot 10^{-13}\,[A]$, $A = 0.1\,\left[mm^2\right]$, $\tau_T = 2 \cdot 10^{-6}\,[s]$ (weighted average transit time).

- Draw the small-signal equivalent circuit of the diode.
- Calculate the small-signal admittance g_d, the depletion capacitance C_j and the diffusion capacitance C_d at the DC working points $V_D = 0.3\,[V]$ and $V_D = -2\,[V]$.
- In which of the two operating points is best to bias the diode to realize a variable capacitor? Why?

Exercise 07

Consider an NPN BJT with parameters: $N_{dE} = 10^{17}\left[cm^{-3}\right]$, $N_{aB} = 10^{16}\left[cm^{-3}\right]$, $D_n = 27\left[cm^2/s\right]$, $D_p = 9\left[cm^2/s\right]$, $\mu_{nE} = 900\left[cm^2V^{-1}s^{-1}\right]$, $A_E = 100\left[um^2\right]$.

- Design the emitter width W_E to have an emitter resistance $R_E = 5\,[\Omega]$.
- The minimum base width achievable with this technology is $W_B = 300\,[nm]$. Calculate the current gain β_F. Assume that $W_B << L_{nB}$ and $W_E << L_{pE}$ and that we can neglect the width of the depletion region of the B-E junction.
- The BJT has $\tau_{n0} = \tau_{p0} = 5 \cdot 10^{-7}\,[s]$. Are the short base and emitter assumptions verified?

Exercise 08

Consider a planar MOSFET structure with a $10 \times 10\left[\mu m^2\right]$ aluminum gate on a p-doped Si substrate with $N_a = 10^{15}\left[cm^{-3}\right]$ at $300\,[K]$. Let us first focus on the gate stack. Upon acquiring a capacitance-voltage (C-V) curve at high frequency, the capacitance of the MOS capacitor in accumulation is measured to be $5.0 \cdot 10^{-13}\,[F]$, and drops to $1.2 \cdot 10^{-14}\,[F]$ in inversion.

- Calculate the thickness of SiO_2 and of the depletion region.
- The C-V plot also shows a flat-band voltage $V_{FB} = -2.0\ [V]$: calculate the interfacial charge density.
- Based on the data obtained in the previous questions, calculate the threshold voltage V_T of this transistor.
- Assume you are operating the transistor in linear regime, at $V_{GS} = V_{th} + 0.10\,[V]$ and $V_{DS} = 10\,[mV]$. Assume an electron mobility of $1.3{\cdot}10^3\left[cm^2 \cdot V^{-1} \cdot s^{-1}\right]$. Calculate the current I_D.

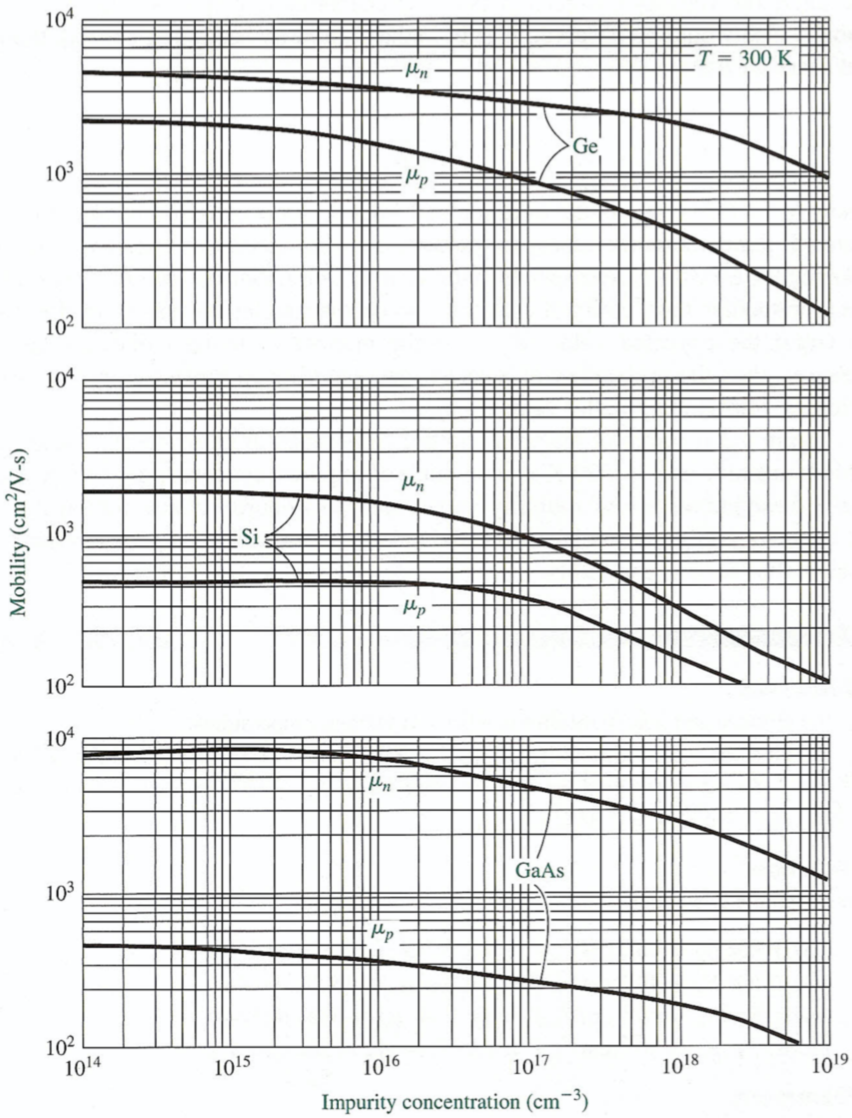


Figure 1: Electron and hole mobilities versus impurity concentrations for germanium, silicon, and gallium arsenide at $T = 300\,[K]$.

Solutions

Exercise 01

- Using Boltzmann approximation and the values given in the preamble:

$$n_i^2 = 4.6 \cdot 10^{19} \cdot 2.3 \cdot 10^{18} \cdot e^{-\frac{3.39}{8.62 \cdot 10^{-5} \cdot 300}} = 1.2 \cdot 10^{-19} \ \left[cm^{-6}\right] \quad (2)$$

$$n_i = \sqrt{1.2 \cdot 10^{-19}} = 3.5 \cdot 10^{-10} \ \left[cm^{-3}\right] \quad (3)$$

- Starting by converting Celsius to Kelvin ($600\ [°C] + 273 = 873\ K$) and using the given equation for the bandgap:

$$E_g = 3.47 - 7.7 \cdot 10^{-4} \cdot \frac{873^2}{873 + 600} = 3.07 \ [eV] \quad (4)$$

  We then use the same equation of the previous point, modifying temperature and band gap in the exponential and accounting for the change in effective density of states by multiplying by the temperature ratio with the correct exponential:

$$n_i^2 = 4.6 \cdot 10^{19} \cdot 2.3 \cdot 10^{18} \cdot \left(\frac{873}{300}\right)^3 \cdot e^{-\frac{3.07}{8.62 \cdot 10^{-5} \cdot 873}} = 5.0 \cdot 10^{21} \ \left[cm^{-6}\right] \quad (5)$$

$$n_i = \sqrt{5.0 \cdot 10^{21}} = 7.1 \cdot 10^{10} \ \left[cm^{-3}\right] \quad (6)$$

  The intrinsic carrier density of GaN at 600 $°C$ is higher (but in the same order of magnitude) than the one of Si at room temperature.

- For example, high temperature or high power electronics.

Exercise 02

- See figure 2.
- From the Boltzmann approximation:

$$p_0 = 1.04 \cdot 10^{19} \cdot e^{-\frac{(10+45) \cdot 10^{-3}}{8.62 \cdot 10^{-5} \cdot 300}} = 1.24 \cdot 10^{18} \ \left[cm^{-3}\right] \quad (7)$$

  From the Fermi-Dirac formula:

$$p_0 = 1.04 \cdot 10^{19} \cdot \frac{1}{1 + e^{\frac{(10+45) \cdot 10^{-3}}{8.62 \cdot 10^{-5} \cdot 300}}} = 1.11 \cdot 10^{18} \ \left[cm^{-3}\right] \quad (8)$$

  And finally:

$$\frac{Boltzmann - FD}{FD} = \frac{1.24 - 1.11}{1.11} = 0.117 \quad (9)$$

$$error_{Boltzmann} = +11.7\% \quad (10)$$

- $55\ meV < 3k_BT$ with $T = 300\ [K]$.

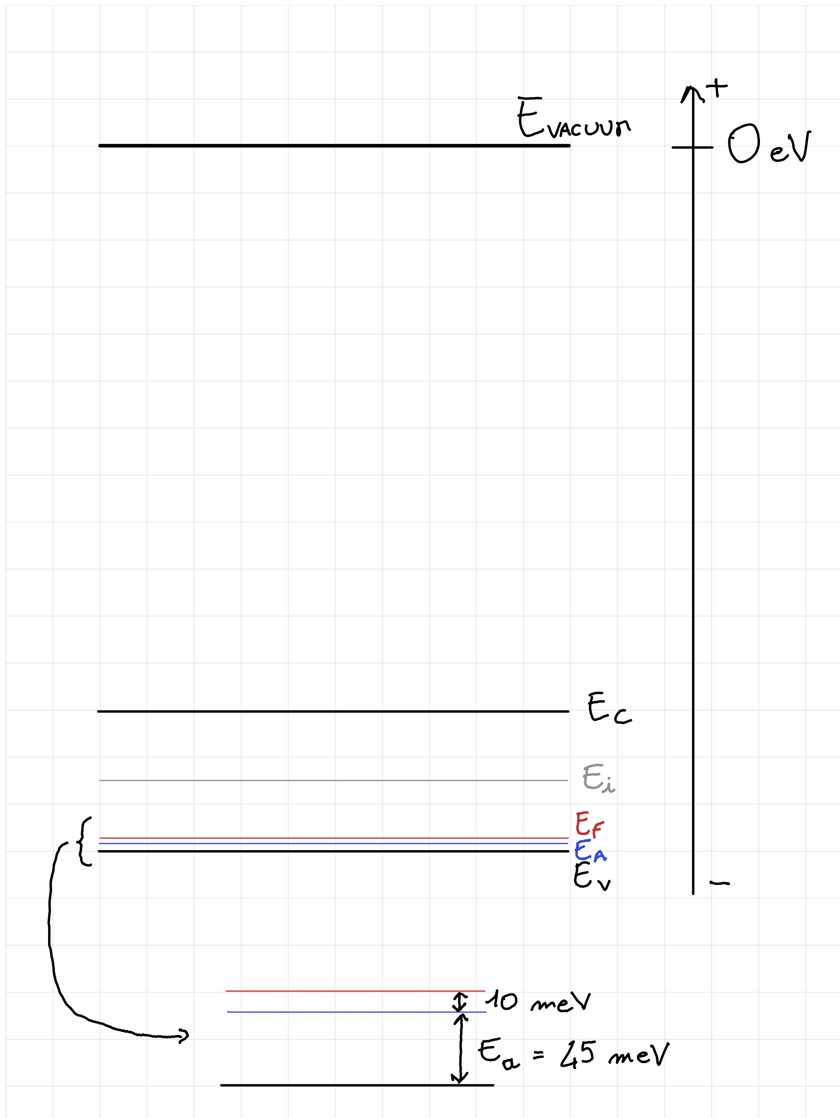


Figure 2: Band diagram.

Exercise 03

- $$R = \frac{V}{I} = \frac{4}{1 \cdot 10^{-3}} = 4 \cdot 10^{3} \ \ [\Omega] \tag{11}$$

$$\rho = R \cdot \frac{A}{L} = 4 \cdot 10^3 \cdot \frac{1 \cdot 10^{-2}}{1} = 40 \ [\Omega \cdot cm] \tag{12}$$

$$\sigma = \frac{1}{\rho} = 2.5 \cdot 10^{-2} \ \left[S \cdot cm^{-1}\right] \tag{13}$$

$$\sigma = q \cdot \mu_p \cdot p \quad \Rightarrow \quad 2.5 \cdot 10^{-2} = 1.6 \cdot 10^{-19} \cdot \mu_p \cdot p \tag{14}$$

By looking at figure 1, we observe that for low doping levels the hole mobility is approximately constant and equal to $4.3 \cdot 10^2 \ \left[cm^2 \cdot V^{-1} \cdot s^{-1}\right]$. Assuming that $p = [N_a^+] = [N_a]$:

$$p = \frac{2.5 \cdot 10^{-2}}{1.6 \cdot 10^{-19} \cdot 4.3 \cdot 10^2} = 3.6 \cdot 10^{14} \ \left[cm^{-3}\right] \tag{15}$$

- We can observe that the hole mobility has approximately the same value up to $[N_a] = 10^{16} \ \left[cm^{-3}\right]$. Since $I \propto \sigma \propto p = [N_a]$ (at $T = 300 \ [K]$), we can easily increase the current by one order of magnitude by increasing the doping concentration $[N_a]$ to $3.6 \cdot 10^{15} \ \left[cm^{-3}\right]$.
  The alternative is to obtain a compensated semiconductor by introducing an n-doping with P so that the condition is satisfied. In order to find $[N_d]$, we proceed iteratively using the formula $\sigma = q \cdot \mu_n \cdot n$, assuming that in this case the conduction is completely dominated by electrons and remembering that $n \approx [N_d] - [N_a]$, where $[N_a] = 3.6 \cdot 10^{14} \ \left[cm^{-3}\right]$ and $\sigma = 2.5 \cdot 10^{-1} \ \left[S \cdot cm^{-1}\right]$.

$$2.5 \cdot 10^{-1} = 1.6 \cdot 10^{-19} \cdot \mu_n \cdot \left([N_d] - 3.6 \cdot 10^{14}\right) \tag{16}$$

$$1.6 \cdot 10^{18} = \mu_n \cdot \left([N_d] - 3.6 \cdot 10^{14}\right) \tag{17}$$

The equation is satisfied for $[N_d] \approx 1.25 \cdot 10^{15} \ \left[cm^{-3}\right]$, which correspond to $\mu_n \approx 1.8 \cdot 10^3 \ \left[cm^2 \cdot V^{-1} \cdot s^{-1}\right]$.

Exercise 04

$$\phi_b = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.717 \, [V] \tag{18}$$

$$x_n\left(V_D\right) = \sqrt{\frac{2\epsilon_{Si} \ N_a}{q\left(N_a + N_d\right) \ N_d} \left(\phi_b - V_D\right)} \tag{19}$$

$$x_p\left(V_D\right) = \sqrt{\frac{2\epsilon_{Si} \ N_d}{q\left(N_a + N_d\right) \ N_a} \left(\phi_b - V_D\right)} \tag{20}$$

Beware that the results in your calculator are in $[cm]$, not in $[m]$!
At thermal equilibrium: $x_{n0} \approx 81 \, [nm]$ and $x_{p0} \approx 303 \, [nm]$.
At $V_D = 0.5 \ V$: $x_n \approx 45 \, [nm]$ and $x_p \approx 167 \, [nm]$ ($\gamma = \sqrt{1 - \frac{V_D}{\phi_b}} \approx 0.55$).

At $V_D = -1\ V$: $x_n \approx 126\,[nm]$ and $x_p \approx 470\,[nm]$ ($\gamma = \sqrt{1 - \frac{V_D}{\phi_b}} \approx 1.55$).
The space charge density $\rho(x)$ is:

$$\rho\left(x\right) = \begin{cases} 0 & , \quad x \in \left]\leftarrow; -x_p\right] \\ -qN_a & , \quad x \in \left]-x_p; 0\right] \\ qN_d & , \quad x \in \left]0; x_n\right] \\ 0 & , \quad x \in \left]x_n; \rightarrow\right] \end{cases} \tag{21}$$

The plot of $\rho(x)$ is shown in Figure 3a. It does not have to be scaled necessarily. However, it is important to show: 1) whether $x_n > x_p$ or vice versa; 2) whether $qN_d > qN_a$ or vice versa; 3) how forward and reverse bias affect x_n and x_p.
The electric field $E(x)$ is:

$$E\left(x\right) = \begin{cases} 0 & , \quad x \in \left]\leftarrow; -x_p\right] \\ -\frac{qN_a}{\epsilon_{Si}}\left(x + x_p\right) & , \quad x \in \left]-x_p; 0\right] \\ \frac{qN_d}{\epsilon_{Si}}\left(x - x_n\right) & , \quad x \in \left]0; x_n\right] \\ 0 & , \quad x \in \left]x_n; \rightarrow\right] \end{cases} \tag{22}$$

The plot of $E(x)$ is shown in Figure 3b. Similarly to $\rho(x)$, when plotting $E(x)$ it is important to show: 1) whether $x_n > x_p$ or vice versa; 2) how forward and reverse bias affect x_n, x_p and E_{max}.
The maximum electric field is given by: $E_{max} = -\frac{qN_d}{\epsilon_{Si}} x_n = -\frac{qN_a}{\epsilon_{Si}} x_p$.
At thermal equilibrium: $E_{max} = -3.75 \cdot 10^4\ V/cm$.
At $V_D = 0.5\ V$: $E_{max} = -2.09 \cdot 10^4\ V/cm$.
At $V_D = -1\ V$: $E_{max} = -5.81 \cdot 10^4\ V/cm$.

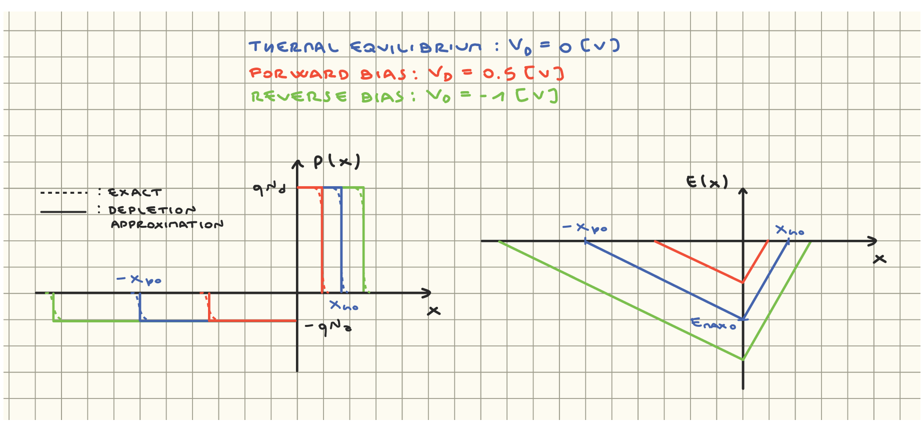


Figure 3: a) $\rho(x)$; b) $E(x)$.

Exercise 05

We calculate the minority carriers diffusion lengths:

$$L_n = \sqrt{D_n \tau_{n0}} \approx 16.4 \, [um] \tag{23}$$

$$L_p = \sqrt{D_p \tau_{p0}} \approx 10.5 \, [um] \tag{24}$$

The long neutral sides conditions hold: $W_p - x_p >> L_n$ and $W_n - x_n >> L_p$. The excess minority carriers concentrations $n'(x)$ and $p'(x)$ follow an exponential decay with x in the quasi-neutral regions, since recombination in the bulk is dominant. We use the corresponding formula for I_S:

$$I_S = Aqn_i^2 \left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d} \right) \tag{25}$$

We calculate the intrinsic carrier concentration n_i of Si at $T_2 = 250 \ K$. We recall that:

$$n_i^2 = N_c N_v exp \left(\frac{-E_g}{kT} \right) \tag{26}$$

Since $N_c \cdot N_v \propto T^3$, we can write the following proportionality:

$$N_c \cdot N_v(300K) : 300^3 = N_c \cdot N_v(250K) : 250^3 \tag{27}$$

that gives $N_c \cdot N_v(250K) = 1.69 \cdot 10^{38} \left[cm^{-6}\right]$. Therefore, using formula (26) we obtain:

$$n_i^2(250K) \approx 4.66 \cdot 10^{15} \left[cm^{-6}\right] \tag{28}$$

($n_i \approx 6.83 \cdot 10^7 \left[cm^{-3}\right]$). Be careful that also kT is different from the room temperature value in the previous calculation.
Now we can calculate I_S at the two temperatures.
At $T_1 = 300 \ K$: $I_S \approx 8.67 \cdot 10^{-13} \, [A]$.
At $T_2 = 250 \ K$: $I_S \approx 1.79 \cdot 10^{-17} \, [A]$.
We calculate the currents at $V_D = 0.5 \ V$ (be careful to kT in this case too):

$$I = I_S \left[exp \left(\frac{V_D}{V_T} \right) - 1 \right] \tag{29}$$

At $T_1 = 300 \ K$: $I(0.5V) \approx 2.14 \cdot 10^{-4} \, [A]$.
At $T_2 = 250 \ K$: $I(0.5V) \approx 2.11 \cdot 10^{-7} \, [A]$.
The $log|I(V)|$ curves at T_1 and T_2 are shown in Figure 4. Two main properties change when changing the temperature: 1) the reverse saturation current I_S; 2) the slope of the exponential, therefore the small-signal admittance $g_d = \frac{\partial i}{\partial v} = \frac{q(I+I_S)}{kT}$.

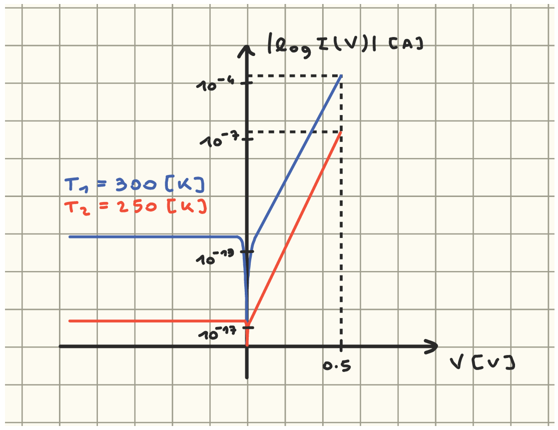
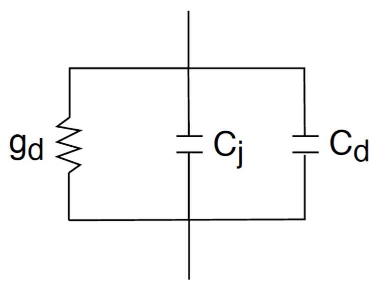


Figure 4: $log|I(V)|$ curves of the PN diode at $T_1 = 300\ K$ and $T_2 = 250\ K$.

Exercise 06

The small-signal equivalent circuit of the diode is:



We know that:

$$\phi_b = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.575\,[V] \tag{30}$$

$$I = I_S\left[exp\left(\frac{V_D}{V_T}\right) - 1\right] \tag{31}$$

that leads $I(0.3V) = 2.17 \cdot 10^{-8}\,[A]$ and $I(-2V) = -I_S = -2.0 \cdot 10^{-13}\,[A]$. The formulas for g_d, C_j and C_d are:

$$g_d = \frac{q(I + I_S)}{kT} \tag{32}$$

$$C_j = A \cdot \frac{\epsilon_{Si}}{x_d} = A \cdot \sqrt{\frac{q\epsilon_{Si} N_a N_d}{2\left(\phi_b - V_D\right)\left(N_a + N_d\right)}} = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\phi_b}}} \tag{33}$$

$$C_d = \frac{q}{kT}\tau_T I \tag{34}$$

Therefore, we can already calculate the small-signal parameters in the two working points. Be careful to include the device section A in the formula for C_j, to get the actual device capacitance (in $[F]$) and not the capacitance density (in $\left[F/cm^2\right]$).
At $V_D = 0.3V$: $g_d = 8.4 \cdot 10^{-7}\,[S]$, $C_j = 12.3\,[pF]$ and $C_d = 1.7\,[pF]$.
At $V_D = -2V$: $g_d = 0\,[S]$, $C_j = 4.0\,[pF]$ and $C_d = 0\,[F]$.
To realize a variable capacitor, the small-signal admittance g_d of the diode must be negligible, so that its small-signal equivalent circuit reduces to a variable capacitance. Since $g_d \propto I + I_S$, it must be $I \approx -I_S$. This condition is verified in a DC working point in reverse bias. Therefore, we choose the working point $V_D = -2V$.

Exercise 07

The resistance of the emitter is calculated as:

$$R_E = \rho\frac{W_E}{A_E} = \frac{1}{q\mu_{nE}N_{dE}}\frac{W_E}{A_E} \tag{35}$$

where $n_0 = N_{dE} >> p_0 = \frac{n_i^2}{N_{dE}}$ in the emitter. Substituting $R_E = 5\,[\Omega]$, we obtain $W_E = 720\,[nm]$.
We then calculate the BJT current gain, with the formula accounting for short base and emitter and neglecting the B-E depletion region:

$$\beta_F = \frac{I_C}{I_B} = \frac{N_{dE}D_nW_E}{N_{aB}D_pW_B} = 72 \tag{36}$$

For the last point we just need to calculate the minority carriers diffusion lengths in the emitter and in the base:

$$L_{nB} = \sqrt{D_n\tau_{n0}} \approx 36.7\,[um] >> W_B \tag{37}$$

$$L_{pE} = \sqrt{D_p\tau_{p0}} \approx 21.2\,[um] >> W_E \tag{38}$$

The short base and emitter conditions are verified.

Exercise 08

- The oxide capacitance is the capacitance in accumulation:

$$\begin{aligned} C_{acc} = C_{ox} = 5.0 \cdot 10^{-13} &= \frac{\epsilon_0 \cdot \epsilon_{r,SiO_2} \cdot A}{t} \\ &= \frac{8.85 \cdot 10^{-14} \cdot 3.9 \cdot 100 \cdot 10^{-8}}{t\ [cm]} \end{aligned} \tag{39}$$

$$t = \frac{8.85 \cdot 10^{-14} \cdot 3.9 \cdot 100 \cdot 10^{-8}}{5.0 \cdot 10^{-13}} = 6.9 \cdot 10^{-7}\ [cm] = 6.9\ [nm] \tag{40}$$

We assume that in inversion the width of the depletion region doesn't change with respect to threshold.
By considering that the capacitance in inversion is the series of the oxide capacitance and the Si capacitance (*method 1*):

$$\frac{1}{C_{inv}} = \frac{1}{C_{tot}} = \frac{1}{C_{Si}} + \frac{1}{C_{ox}} \quad \Rightarrow \quad \frac{1}{C_{Si}} = \frac{1}{C_{tot}} - \frac{1}{C_{ox}} \tag{41}$$

$$\frac{1}{C_{Si}} = \frac{1}{1.2 \cdot 10^{-14}} - \frac{1}{5.0 \cdot 10^{-13}} = 8.1 \cdot 10^{13}\ \left[F^{-1}\right] \tag{42}$$

$$C_{Si} = \frac{1}{8.1 \cdot 10^{13}} = 1.2 \cdot 10^{-14} = \frac{\epsilon_{Si} \cdot A}{x_d} \tag{43}$$

$$x_d = \frac{8.85 \cdot 10^{-14} \cdot 11.7 \cdot 100 \cdot 10^{-8}}{1.2 \cdot 10^{-14}} = 8.6 \cdot 10^{-5}\ [cm] = 0.86\ [\mu m] \tag{44}$$

By using the relationship between ϕ_p and $x_{d_{th}}$ (*method 2*):

$$\begin{aligned} \phi_p &= -\frac{kT}{q} ln\left(\frac{N_a}{N_i}\right) = -\frac{1.38 \cdot 10^{-23} \cdot 300}{1.60 \cdot 10^{-19}} \cdot ln\left(\frac{10^{15}}{1.5 \cdot 10^{10}}\right) \\ &= -287\ [mV] \end{aligned} \tag{45}$$

$$\begin{aligned} x_{d_{th}} &= \sqrt{-\frac{2 \cdot 2\phi_b \cdot \epsilon_{Si}}{q \cdot N_a}} = \sqrt{\frac{2 \cdot 0.574 \cdot 8.85 \cdot 10^{-14} \cdot 11.7}{1.60 \cdot 10^{-19} \cdot 10^{15}}} \\ &= 8.62 \cdot 10^{-5}\ [cm] \end{aligned} \tag{46}$$

- From the given equation and constants:

$$V_{FB} = \phi_{ms} - \frac{q \cdot Q_{ss}}{C_{ox}} \quad (\textit{capacitance per unit area!!}) \tag{47}$$

$$\begin{aligned} \phi_{ms} &= \phi_{m,\ Al} - \left(\chi'_{Si} + \frac{E_{g,\ Si}}{2q} - \phi_p\right) \\ &= 3.2 - (3.25 + 0.560 + 0.287) \\ &= -897\ [mV] \end{aligned} \tag{48}$$

$$\begin{aligned} Q_{ss} &= \frac{\phi_{ms} - V_{FB}}{q} \cdot C_{ox} = \frac{-0.897 + 2.0}{1.60 \cdot 10^{-19}} \cdot \frac{5.0 \cdot 10^{-13}}{100 \cdot 10^{-8}} \\ &= 3.4 \cdot 10^{12}\ \left[cm^{-2}\right] \end{aligned} \tag{49}$$

- Using the given formulas:

$$\begin{aligned} \gamma &= \frac{\sqrt{2\epsilon_{Si} q N_a}}{C_{ox}} \\ &= \frac{100 \cdot 10^{-8}}{5.0 \cdot 10^{-13}} \cdot \sqrt{2 \cdot 8.85 \cdot 10^{-14} \cdot 11.7 \cdot 1.60 \cdot 10^{-19} \cdot 10^{15}} \\ &= 3.6 \cdot 10^{-2}\ \left[V^{1/2}\right] \end{aligned} \tag{50}$$

$$V_{th} = V_{FB} - 2\phi_p + \gamma\sqrt{-2\phi_p} = -2.0 + 0.574 + 3.6 \cdot 10^{-2} \cdot \sqrt{0.574}$$
$$= -1.4 \ [V] \tag{51}$$

- By taking the given formula and plugging the given values (because of the subtraction in the parenthesis, no data from the previous questions is needed for this point):

$$\begin{aligned} I_D &= \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - \frac{V_{DS}}{2} - V_{th}\right)V_{DS} \\ &= 1 \cdot 1.3 \cdot 10^3 \cdot \frac{5.0 \cdot 10^{-13}}{100 \cdot 10^{-8}}(0.10 - 0.0050) \cdot 0.010 \\ &= 6.2 \cdot 10^{-7} \ [A] \end{aligned} \tag{52}$$